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Stress analysis of finite composite laminate with multiple loaded holes

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Abstract

Based on the classical laminated plate theory, a finite composite plate weakened by multiple elliptical holes is treated as an anisotropic multiple connected plate. Using the complex potential method in the plane theory of elasticity of an anisotropic body, an analytical study concerned with the stress distributions around multiple loaded holes in finite composite laminated plates subjected to arbitrary loads was performed. The analysis makes use of the Faber series expansion, conformal mapping and the least squares boundary collocation techniques. The effects of plate and hole sizes, layups, the relative distance between holes, the total number of holes and their locations on the stress distribution are studied in detail. Some conclusions are drawn. © 1998 Elsevier Science Ltd. All rights reserved.

1. Introduction

It is well known that holes and cutouts cause serious problems of stress concentrations due to the geometry discontinuity. These problems are even more serious in structures made of composite materials since the materials exhibit anisotropic behaviour, and the structures are more sensitive to stress concentrations due to its brittle behaviour. Therefore, more attentions have been paid by many researchers to the stress concentration of composite structures. The closed form solution to stress concentration around a circular hole in an infinite orthotropic plate was first obtained by Lekhnitskii (1957) and Savin (1961) using the complex potential method. Jong (1977), Zhang and Ueng (1984) and Hyer and Klang (1987) studied the stress distribution of infinite composite laminates with a pin hole. It is very difficult to analyze the stress state of a laminated plate weakened by multiple holes because it is a multiple connected boundary value problem. The stress states of an infinite orthotropic plate weakened by two elliptical holes with parallel axes was studied by Kosmodamianskii and Chernic (1981). Lin and Ueng (1987) studied a similar problem. Extensive

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studies have been made by the author and their colleagues (Xu and Fan 1991; Xu et al., 1992a; Xu, 1992b; Xu and Fan, 1993a, b; Fan and Wu, 1988) on the thermoelasticity problem of infinite composite laminates. However, the analytical solutions to stress concentration of a finite plate with multiple holes have not been found yet. Gerhardt (1984) obtained the solution to a finite plane weakened by a circular hole using the hybrid finite element method. Similar problems were studied by Ogonowski (1980) and Lin and Ko (1988) by using the boundary collocation approach. These methods, however, still suffered some drawbacks, such as large data preparations, long cpu time and low accuracy. Recently, Xu (1992b) and Xu et al. (1995a, b, c) studied extensively the stress concentration of a finite laminated plate with multiple unloaded elliptical holes using conformal mapping, the Faber series expansion and the least squares boundary collocation. According to the author's knowledge, there are no solution to stress distributions of a finite plate with multiple loaded holes in the literature. This is an important problem because mechanically multi-fastened joints are widely used in composite structures. Thus, the objective of this paper is to obtain a solution to the title problem.

In this paper, an analytical study concerned with the stress distributions around multiple loaded holes in a finite composite laminated plate was performed by using the complex potential method in the plane theory of elasticity of an anisotropic body with the aid of the Faber series expansion, conformal mapping and the least squares boundary collocation techniques. Furthermore, the effects of various parameters are studied in detail, these include plate and hole sizes, layups, the relative distance between holes, the total number of holes and their locations. Finally some important conclusions are drawn.

2. Basic equations

In the classical theory, the laminated composite plate is treated as an anisotropic plate. Therefore, the constitutive equation for a laminated plate in plane stress (Jones, 1975) is

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_y \\ \bar{\sigma}_{xy} \end{Bmatrix} \tag{1}$$

where $\{\bar{\sigma}\}$ is the average in-plane stress and a_{ij} is the equivalent compliance coefficients depending on the fiber orientation, the stacking sequence and the property of each lamina.

In a rectangular coordinate system x_i ($i = 1, 2, 3$), let u_i, σ_{ij} and ε_{ij} be the displacement, stress and strain, respectively. The basic equations of theory of elasticity body are

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i})$$

$$\sigma_{ij} = E_{ijkl}\varepsilon_{kl}\sigma_{ij,j} = 0 \tag{2}$$

where E_{ijkl} is the elasticity coefficient, u_i, σ_{ij} and ε_{ij} are independent of x_3 coordinate for the plane problem. By introducing the Airy stress function, the solutions of eqn (2) can be expressed as (Lekhnitskii, 1957)

$$\sigma_x = 2Re\sum_{j=1}^{2}\mu_j^2\varphi_j'(z_j)$$

$$\sigma_y = 2Re\sum_{j=1}^{2}\varphi_j'(z_j)$$

$$\tau_{xy} = -2Re\sum_{j=1}^{2}\mu_j\varphi(z_j) \tag{3}$$

$$u = 2Re\sum_{j=1}^{2}p_j\varphi_j(z_j)-\omega y+u_0$$

$$v = 2Re\sum_{j=1}^{2}q_j\varphi_j(z_j)+\omega x+v_0 \tag{4}$$

where $z_j = x+\mu_j y$, μ_j is the root of the characteristic equation [eqn (5)], a complex parameter represents the anisotropic extent of the laminated plate.

$$a_{11}\mu^4-2a_{16}\mu^3+(2a_{12}+a_{66})\mu^2-2a_{26}\mu+a_{22} = 0 \tag{5}$$

where

$$p_j = a_{11}\mu_j^2+a_{12}-a_{16}\mu_j$$

$$q_j = a_{12}\mu_j+\frac{a_{22}}{\mu_j}-a_{26} \tag{6}$$

$\varphi_j(z_j)$ is an analytic function in the generalized region S_j obtained by the affine transformation $z_j = x+\mu_j y$ from the physical region S.

When the forces X_n and Y_n are prescribed on the boundary, the boundary conditions are of the form

$$2Re\sum_{j=1}^{2}\varphi_j(z_j) = \mp\int_0^t Y_n\,\mathrm{d}s+c_1$$

$$2Re\sum_{j=1}^{2}\mu_j\varphi_j(z_j) = \pm\int_0^t X_n\,\mathrm{d}s+c_2 \tag{7}$$

The upper signs on the right-hand side of eqn (7) are to be taken for the outer contour, and the lower signs for the inner contour, i.e., for the contour of a cutout.

When the displacements u, v are prescribed on the boundary, the boundary conditions are of the form

$$2Re\sum_{j=1}^{2}p_j\varphi_j(z_j) = u-\omega y+u_0$$

$$2Re\sum_{j=1}^{2} q_j\varphi_j(z_j) = v+\omega x+v_0 \tag{8}$$

3. Analysis

Consider a finite composite laminated plate weakened by multiple elliptical holes with contours $L_0, L_1, L_2, \ldots, L_l$, as shown in Fig. 1. Their semi-major, semi-minor axes and centers are a_m, b_m, and z_m $(m = 1, 2, \ldots l)$, respectively. By affine transformation $z_j = x+\mu_j y$ from the region S onto the region S_j, the point z_m in the region S is corresponding to the point z_{jm} in the region S_j.

If no body force is applied in the plate, the complex potential function $\varphi_j\ (z_j)$ can be expressed as (Xu, 1992b)

$$\varphi(z_j) = \sum_{m=1}^{l} A_{jm}\ln(z_j-z_{jm})+\varphi_{0j}(z_j)+\sum_{k=0}^{\infty} b_{jk}P_k(z_j) \quad (j = 1, 2) \tag{9}$$

where $\varphi_{0j}\ (z_j)$ is a holomorphic function in the infinite region with l elliptical holes. $P_k(z_j)$ is the Faber polynomial of the region bounded by the contour L_{j0}.

The mapping function is given as follows:

$$z_j-z_{jm} = R_{jm}\left(\xi_{jm}+\frac{t_{jm}}{\xi_{jm}}\right)(m = 1, 2, \ldots, l, j = 1, 2) \tag{10}$$

where

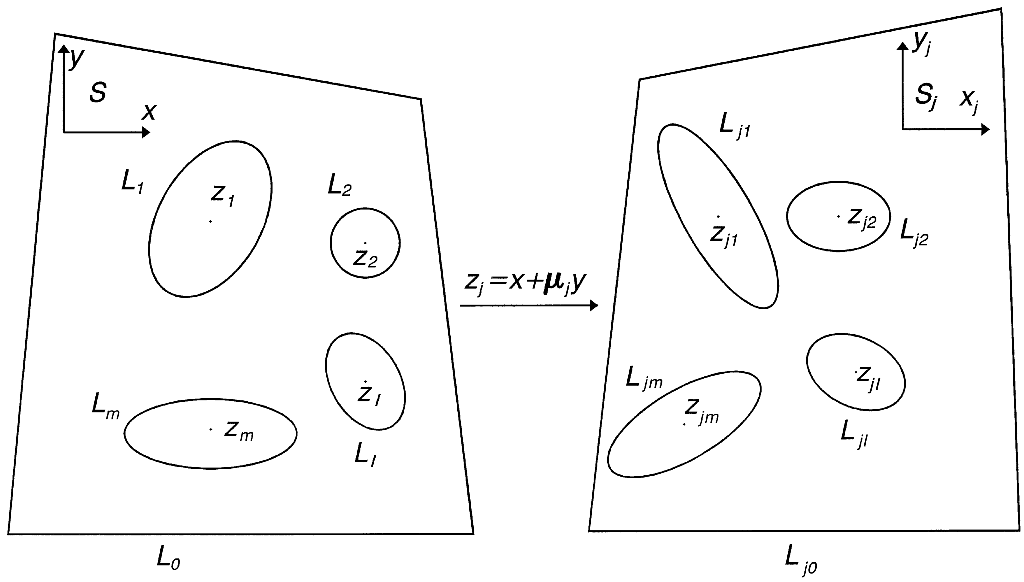

Fig. 1. A finite composite laminated plate weakened by multiple elliptical holes.

$$R_{jm} = \frac{a_m - i\mu_j b_m}{2}, \quad t_{jm} = \frac{a_m + i\mu_j b_m}{a_m - i\mu_j b_m}$$

This mapping function transforms the exterior of the hole m in the complex plane z_j into the exterior of a unit circle, $\xi_{jm} = \exp(i\theta)$, in complex plane ξ_{jm}. If the major and minor axes are not parallel to the coordinate axes, resort to rotation mapping.

Using Laurent series expansion and the Faber polynomial in the general region, the complex potential function can be shown as (Xu, 1992b):

$$\varphi_j(z_j) = \sum_{m=1}^{l}\left(A_{jm}\ln\xi_{jm} + \sum_{k=1}^{\infty}\frac{b_{jmk}}{\xi_{jm}^k}\right) + \sum_{k=0}^{\infty} a_{jk} z_j^k \quad (j = 1, 2) \tag{11}$$

Let the external stress components acting on contour L_m be X_m and Y_m. The coefficients A_{jm} ($j = 1,2; m = 1, 2, \ldots, l$) can be determined by the single-value condition of the displacements (Xu, 1992b).

$$2Re\sum_{j=1}^{2} iA_{jm} = \frac{Y_m}{2\pi}$$

$$2Re\sum_{j=1}^{2} i\mu_j A_{jm} = -\frac{X_m}{2\pi}$$

$$2Re\sum_{j=1}^{2} i\mu_j^2 A_{jm} = -\frac{a_{12}}{a_{11}}\frac{Y_m}{2\pi} - \frac{a_{16}}{a_{11}}\frac{X_m}{2\pi}$$

$$2Re\sum_{j=1}^{2} i\frac{A_{jm}}{\mu_j} = \frac{a_{12}}{a_{22}}\frac{X_m}{2\pi} + \frac{a_{26}}{a_{22}}\frac{Y_m}{2\pi} \tag{12}$$

Obviously, the complex potential functions eqn (11) are analytic in the region S_j. Once the unknown coefficients b_{jmk} and a_{jk} are determined by using the boundary conditions, the stress and displacement fields can be uniquely obtained according to the uniqueness theorem of the theory of elasticity.

Suppose, the external forces X_n, Y_n are applied, or the displacements $u(t)$, $v(t)$ are given on the contour, the boundary conditions eqns (7) and (8) can be expressed as:

$$\sum_{j=1}^{2}[r_j\varphi_j(z_j) + s_j\overline{\varphi_j(z_j)}] = f(t) \tag{13}$$

where

$$r_j = 1 + i\mu_j, \quad s_j = 1 + i\overline{\mu_j}$$

$$f(t) = \pm\int_0^t i(X_n + iY_n)\,ds + c$$

when the surface forces are given. The upper and lower signs correspond to the outer and inner contours, and,

$$r_j = p_j + iq_j, \quad s_j = \overline{p_j} + i\overline{q_j}$$

$$f(t) = u(t) + iv(t) + i(v_0 + \omega x) + (u_0 + \omega y)$$

if the displacements are prescribed on the boundary.

Then, the right hand side of eqn (13) can be expanded into a complex Fourier series, a power series of $\sigma = \exp(i\theta)$.

From the mapping function eqn (10), it can be seen that function $\xi_{jm}(z_j)$ is holomorphic in the complex plane z_j weakened by the hole m. Therefore, functions ξ_{jm}^{-n}, $\ln \xi_{jm}$ are holomorphic in the interior of the p-th ($p \neq m$) hole and are continuous on its boundary. Thus they can be expanded into a Faber series,

$$\xi_{jm}^{-n} = \sum_{k=0}^{\infty} = A_{n,k}^{jm} P_{kp}(z_j)$$

$$ln\, \xi_{jm} = \sum_{k=0}^{\infty} Q_k^{jm} P_{kp}(z_j) \tag{14}$$

Similarly, one has

$$z_j^n = \sum_{k=0}^{\infty} H_{n,k}^j P_{kp}(z_j) \tag{15}$$

where $P_{kp}(z_j)$ is the k-th Faber polynomial for the ellipse L_{jp} in the complex z_j plane, and

$$P_{kp}(z_j) = \xi_{jp}^k + \frac{t_{jp}^k}{\xi_{jp}^k} \quad (k \geqslant 1)$$

$$P_{0p} = 1 \tag{16}$$

where the coefficients $A_{n,k}^{jm}$, Q_k^{jm}, $H_{n,k}^j$ in Faber series can be determined by Fourier expansion method (Xu, 1992b). Substituting eqns (14) and (15) into the complex potential expression eqn (11) and using $\xi_{jp} = \exp(i\theta) = \sigma$ in the contour L_{jp} of the p-th hole yield the boundary values of $\varphi_j(z_j)$ ($j = 1, 2$), which are in a power series of σ.

It is easy to prove (Xu, 1992b) that the point $z = z_m + a_m \cos\theta + ib_m \sin\theta$ on the physical region is transformed into the point $\sigma = \exp(i\theta)$ on the plane ξ_{jm} plane by using the affine transformation $z_j = x + \mu_j y$ and the mapping transformation eqn (10). Taking the values of $\varphi_j(z_j)$ a partial sum up to the N-th power term and substituting them into the boundary condition of every elliptical hole, and equating the coefficients of the same power σ^k ($k = 0, \pm 1, \pm 2, \ldots, \pm N$) on both sides of every equation. The $(2N+1)l$ linear equations about the unknown coefficients b_{jmk}, a_{jk} and C_p ($p = 1, 2, \ldots, l$) can be obtained. But these linear equations obtained from inner boundary conditions are not sufficient to determine all the coefficients. Therefore, it is necessary to use outer boundary conditions. Although for smooth outer contour L_0 of the plate, the accurate solution can be acquired by using the Faber polynomial in the general region, the calculation is lengthy and complicated. A more convenient method, the least squares boundary collocation technique, is therefore used in this paper. Taking the collocation points z_{ck} ($k = 1, 2, \ldots, M, M \geqslant 2N$) along the outer contour L_0 and substituting z_{ck} into the boundary condition of eqn (13), the linear equations about the unknown coefficients b_{jmk} and a_{jk} that satisfy the outer boundary conditions are obtained. These equations, together with $(2N+1)l$ equations that satisfy the inner boundary conditions are used to determine the complex potential function $\varphi_j(z_j)$; the stress and the displacement fields in the laminated plate can be obtained by eqns (3) and (4).

Obviously, the complex potential function $\varphi_j(z_j)$ is an analytic function in the region S_j. Therefore, the accuracy of the solution can be judged according to whether the boundary conditions are satisfied fully or not. In the present method, the inner boundary conditions can be satisfied accurately (absolute error less than 10^{-5}). By increasing the number of collocation points, the outer boundary conditions can also be more fully satisfied to ensure that the relative error is within one percent. As from Saint-Venant principle, much more accurate results of the stress distribution around holes can be obtained by using the present method, and in fact this is the main concern of many researchers.

4. Numerical results

Consider a finite laminated plate with a loaded hole of diameter D, as shown in Fig. 2. The laminated plate is composed by T300/5222. The material properties are

$$E_1 = 126.2\text{ GPa} \quad E_2 = 7.3\text{ GPa} \quad v_{12} = 0.247 \quad G_{12} = 4.5\text{ GPa}$$

The loads applied on the hole are

$$P_r = \frac{4P}{\pi D}\cos\theta \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

$$P_r = 0 \quad \frac{\pi}{2} \leqslant \theta \leqslant \frac{3\pi}{2}$$

$$P_{r\theta} = 0 \tag{17}$$

where p_r and $p_{r\theta}$ are the distribution forces in the radial and the circumferential directions around the hole, respectively. P is principal vector of the force applied on the contour of the hole.

Figure 3 shows the stress distribution, $\sigma_\theta/(P/D)$, around the hole in a laminated plate $(0_4/\pm 45)_s$. The maximum stress occurs at $\theta = 90^\circ$. The results, thus, indicate that the relative size of the plate has significant effects on the stress distribution. The stress concentration increases rapidly with the decrease of the relative width W/D. When $W/D > 10$, it is reasonable to assume that the finite plate can be treated as an infinite plate in most engineering analyses. It is worth mentioning that

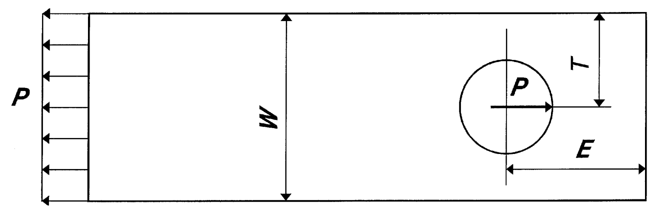

Fig. 2. A finite laminated plate with a loaded hole of diameter D.

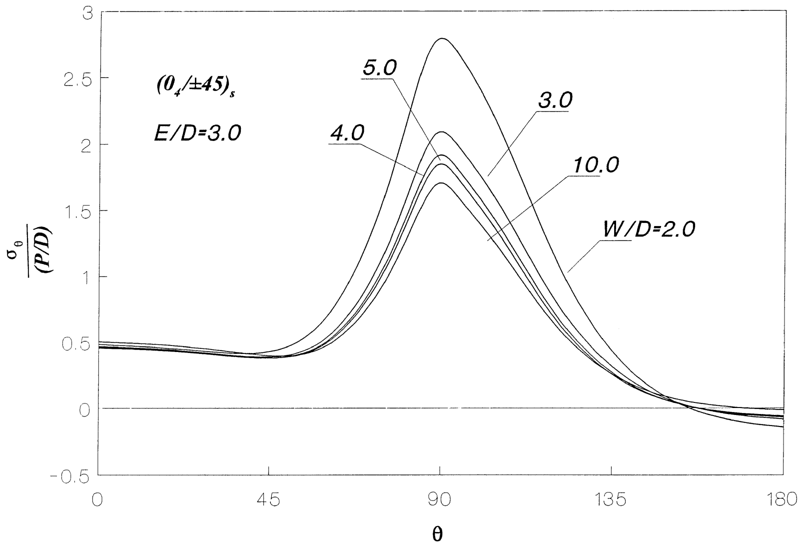

Fig. 3. The stress distribution of a laminated plate with a loaded hole.

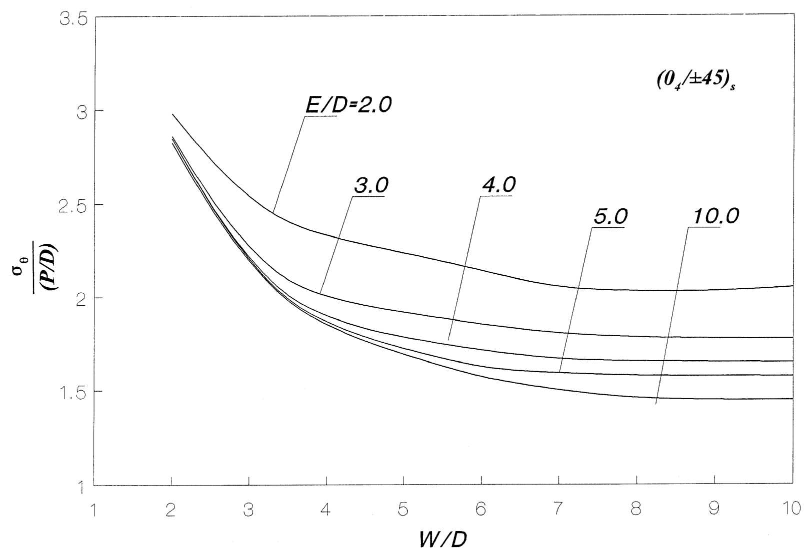

Fig. 4. The effects of the relative plate size W/D and E/D on the stress concentration.

all results are obtained by taking a partial sum up to the 10th power and 32 collocation points on the outer boundary. Numerical results indicate that the accuracy to satisfy boundary conditions has exceeded the requirement mentioned previously. Thus, the present solution has the feature of high accuracy and fast convergence.

The effect of the relative width W/D and relative side distance E/D of the plate on the maximum stress $\sigma_\theta/(P/D)$ $(\theta = 90^\circ)$ around the hole is displayed in Fig. 4. It can be seen from the figure that

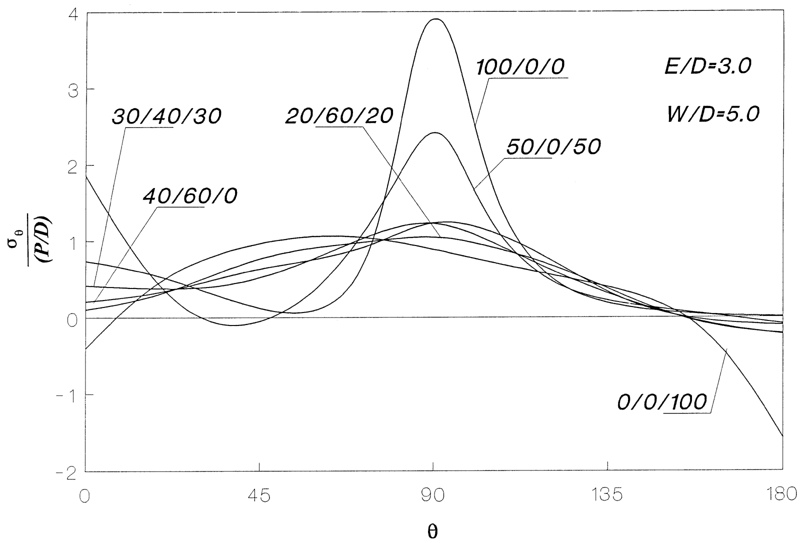

Fig. 5. The effect of layups of laminates on the stress distribution around a hole.

the smaller the relative width W/D and the relative side distance E/D, the more acute the stress concentrations would be. When $W/D > 8$, the maximum stress is almost independent of W/D. However, with the increase of W/D, the effect of E/D on the stress distribution becomes more significant. When $W/D \leqslant 3.0$, the effect of E/D becomes smaller. Numerical results indicate that when $E/D \geqslant 4.0$, $W/D \leqslant 5.0$, the stress concentration is almost independent of E/D because of the simultaneous effects of E/D and W/D.

Figure 5 displays the effect of the layups $(0_\alpha/\pm 45_\beta/90_\gamma)_s$ on the stress $\sigma_\theta/(P/D)$ distribution. The stress distribution around the hole strongly depends on the percentage of each layup in the whole plate. In general, to increase the number of the $\pm 45^\circ$ lamina is beneficial in decreasing the stress concentration because it reduces the extent of the anisotropy of the laminates. For the laminated plate (0/0/100), the maximum stress occurs at $\theta = 180^\circ$ because circumferential stiffness at $\theta = 180^\circ$ is much larger then that at $\theta = 90^\circ$, although the circumferential strain at $\theta = 180^\circ$ is smaller than that at $\theta = 90^\circ$.

The following discussions focus on the stress distribution around the holes of the laminated plate with multiple loaded holes, as shown in Fig. 6 and 7. Each hole is applied a force with a principal force vector of P and is in the form of eqn (17).

Figure 8 shows the stress distribution around the holes of the plate with two holes in series (Fig. 6a). The effect of the relative center-to-center distance l/D between two holes is easily seen. When the relative center-to-center distance l/D becomes smaller, the stress concentration increases rapidly. When $l/D \geqslant 4.5$, the effect of the relative center-to-center distance on the stress distribution is negligible. Comparing with the result of the single hole, however, the stress concentration of hole 2 is still approximately 5.7% lower. This is due to the existence of hole 1 causing nonuniform stress distribution in the region between the two holes. For hole 1, the load around the edge of hole 2 is the pass-load. Thus, the pass-load and its own applied load together cause a much more serious stress concentration in hole 1.

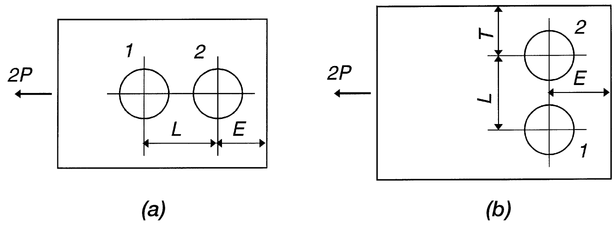

Fig. 6. A finite laminated plate with two loaded holes.

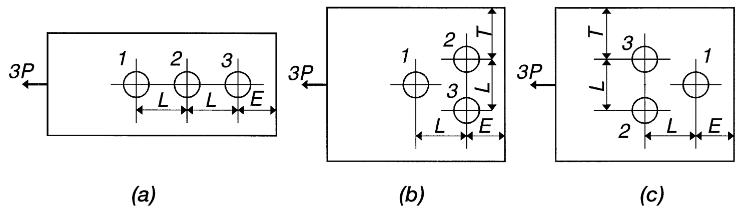

Fig. 7. A finite laminated plate with three loaded holes.

The circumferential stress distribution around the hole where maximum stress occurs for laminated plates with two or three loaded holes (Figs 6 and 7) are shown in Figs 9 and 10, respectively. The effect of the hole arrangements on the stress σ_θ patten around holes is obvious. The arrangements of holes in Fig. 6a for the case of two holes and in Fig. 7c for the case of three holes are beneficial in decreasing the stress concentration. In the case of two holes, which are placed in parallel, as shown in Fig. 6b, the stress concentration would be increased even though there is no pass-load applying to each hole. It can be predicted that the arrangement of holes can have opposite effect on the stress concentration, as shown in Fig. 6b and Fig. 6a when l/D increases to a certain value. In the case of three holes, however, the stress concentration for the arrangement shown in Fig. 7c is lower than that for the arrangement shown in Fig. 7a, because the pass-load of each hole in Fig. 7c is only a quarter of the pass-load of the holes in Fig. 7a.

5. Conclusions

Based on the results reported in this paper, the following conclusions can be drawn.

(1) The stress distribution around the holes depends strongly on the layups of the laminates. In general, increases in the number of the $\pm 45^\circ$ lamina is beneficial in decreasing the stress concentration.

(2) The relative center-to-center distance l/D has significant effects on the stress distribution. In general, the shorter the relative distance, the larger the stress concentration. When $l/D > 4.5$, the effect becomes negligible.

(3) Decrease the relative width W/D of a laminated plate, the stress concentration would increase rapidly. When $W/D > 10$, it is reasonable to assume that the finite plate can be treated as an

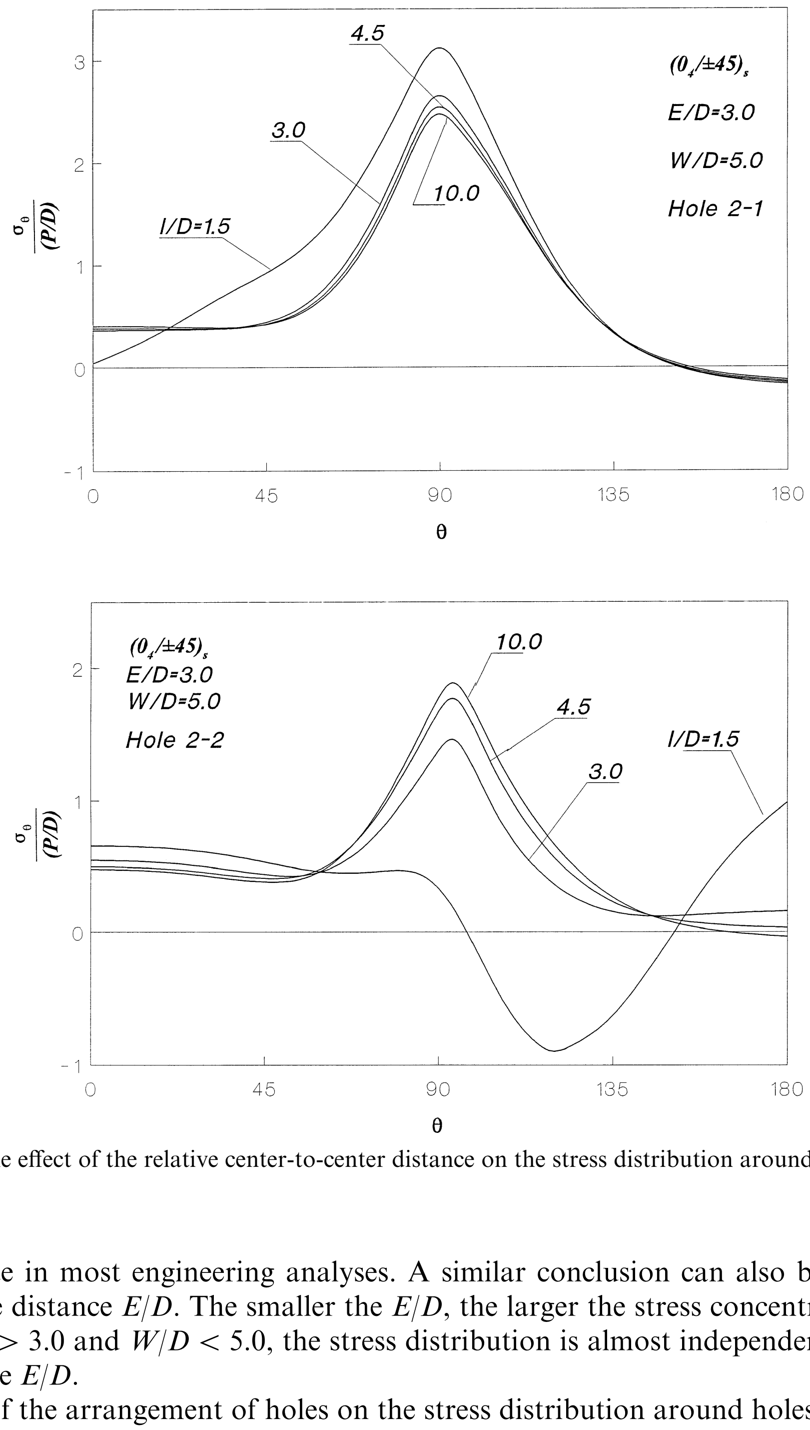

Fig. 8. The effect of the relative center-to-center distance on the stress distribution around the holes.

infinite plate in most engineering analyses. A similar conclusion can also be drawn for the relative side distance E/D. The smaller the E/D, the larger the stress concentration would be. When $E/D > 3.0$ and $W/D < 5.0$, the stress distribution is almost independent of the relative side distance E/D.

(4) The effect of the arrangement of holes on the stress distribution around holes is obvious. The arrangement which causes smooth stiffens change along the direction of the applied force and smaller pass-load for the hole where maximum stress occurs is beneficial in reducing stress concentration.

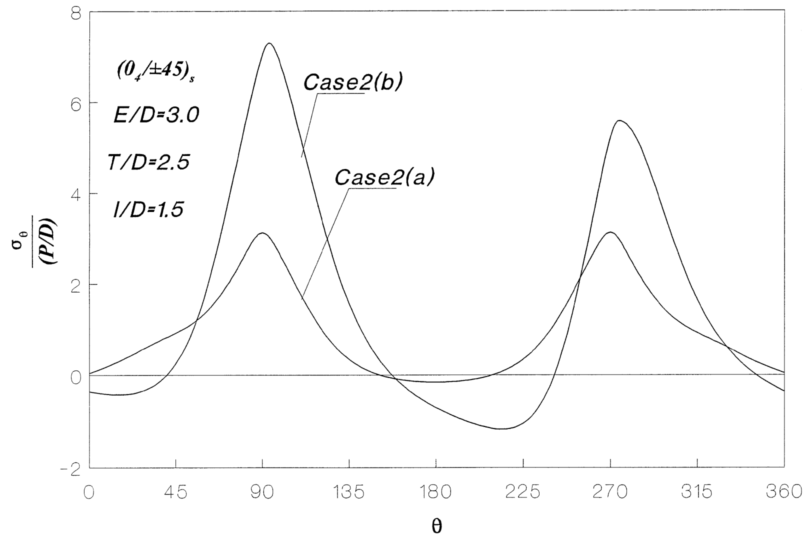

Fig. 9. The effect of the position of the holes on the stress concentration of a laminated plate with two loaded holes.

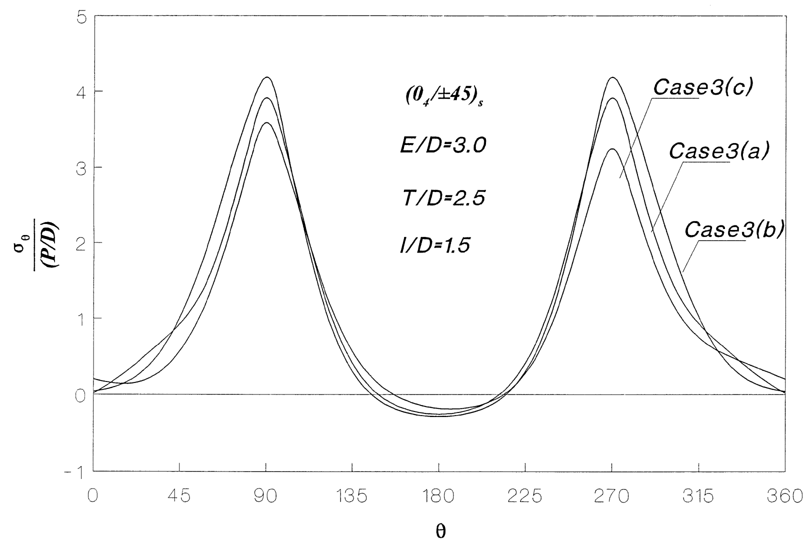

Fig. 10. The effect of the position of the holes on the stress concentration of a laminated plate with three loaded holes.

(5) The present method is very efficient for analysis of the stress distribution of finite composite laminates with multiple loaded holes. It has many advantages such as high accuracy, short computer time and is convenient to use. It can be used to analyze the important problem of multiple bolted joints by introducing compatible displacement conditions.

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